STAT 2593

Lecture 006 - Axioms, Interpretations, and Properties of Probability

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Axioms, Interpretations, and Properties of Probability

Learning Objectives

1. Understand the frequentist interpretation of probability.
2. Understand the three axioms of probability.
3. Understand and manipulate properties arising from the axioms of probability.

What do I **really** mean if I say "the probability of rolling a 1 on a die is 1/6"?

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  - From this vantage point, probability is the long-run proportion of times that an event occurs.
  - If you repeat an experiment many, many, many times, you could count the frequency.
- There are other interpretations of *what* probability is, but we are not concerned with them.

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- These are the only first-order assumptions that we make regarding probability.

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  $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.
- For any three events, A, B, and C,

$$\begin{aligned} P(A \cup B \cup C) = P(A) + P(B) + P(C) \\ -P(A \cap B) - P(A \cap C) \\ +P(A \cap B \cap C). \end{aligned}$$

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- Then, by definition,

$$P(A) = \frac{N_A}{N}.$$

Summary

- Frequentist probabilities refer to long-run proportions on repeated experiments.
- There are three basic axioms of probability (positivity, the unitary property, and additivity).
- Further properties can be derived by manipulating these axioms.
- The value of a probability can be assigned, supposing countable outcomes, as the proportion of outcomes in which the event occurs.